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# Computing Average Skidding Distance for Logging Areas with Irregular Boundaries and Variable Log Density

Dennis M. Donnelly

## Abstract

Current silvicultural practices may result in different cutting intensities on small adjacent areas, resulting in different log densities over the log skidding area. Previous theory on skid distance computation is revised in terms of irregular skid area boundaries and variable log density. A program for a hand-held calculator is presented to compute average skid distance for irregular shaped areas. Methods presented apply either to tractor skidding or to cable yarding distances.

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## Introduction

This report describes a computational routine to figure average skid distance for an irregular area with or without variable log density and presents a method to compute average yarding[2] distance with a hand-held calculator. Theory underlying this computer program is presented in an appendix, as is theory underlying the application of the variable log density formula, but understanding of this theory is not necessary to use the computer program.

Methods currently available for computing average skid distance assume that logs are evenly distributed over the area. While this assumption may have been satisfactory in the past, current timber harvest practice can cause different log densities on subareas of the same skid area. Average skid distance can be significantly different if variable log density is considered by subarea.

## Past Work

Average skid distance has been computed for many years in sale appraisal and logging planning. Matthews (1942) used the method of equal areas to derive average skid distance (ASD) factors for areas shaped like circles, wedges, squares, and rectangles. Other investigators have shown that Matthews' factors for the circle and wedge are in error (Lysons and Mann 1965). They show that average skid distance for these two shapes is two-thirds of the external (maximum possible) skidding distance (ESD), not 0.707 of the external skidding distance as Matthews computed.

Suddarth and Herrick (1964) computed formulas based on integral calculus for average skid distance of areas shaped like circles, wedges, right triangles, and rectangles. Based on their work, Matthews' skid distance factor for a rectangle is also questionable. According to Matthews, ASD = 0.578 (ESD), but Suddarth's formula gives the result, ASD = 0.593 (ESD). This is only a 2 ½% error, which is negligible for most logging applications since other factors affecting skid production introduce more serious variations. However, the difference should be noted.

Unfortunately, skidding areas are not always shaped like circle segments or rectangles. Integral calculus methods using numerical approximations for computing average skid distance have been extended to irregularly shaped areas (Peters and Burke 1972). A desk top computer and digitizer facilitate computation of average skid distance and total area within the prospective skidding area. However, a digitizer is not always readily available, so, the numerical method described here is an efficient substitute which can be applied on a hand-held programable calculator. If this equipment also is unavailable, computations can be done on a desk-top calculator with storage registers, although analysis time will then increase. Appendix 1 presents basic program logic and information.

## Different Log Densities

A yarding area may have two or more distinctly different log densities. How this condition affects average yarding distance (AYD) depends on the density difference and the location of each different log density area with respect to the landing.

A region of heavy log density located far from the landing will significantly increase average yarding distance. The opposite is also true. How much difference in skid distance is involved in these two extremes can be determined using the methods described here.

There are actually two distinct parts to the method presented. The first, the computer program, helps compute average yarding distance for a sub-area where log density is assumed to be uniform. The method is not radically different from those cited

*[2] In this paper, the words yarding and skidding are used interchangeably. The computing methods presented apply either to cable yarding or tractor skidding distance computation. Distance computation for cable yarding, however, does not include lateral yarding distance from the cable road.*

earlier, except for its theoretical base, and no digitizer is required. The second part allows the logging analyst to combine two or more areas of different log densities to find an overall average yarding distance.

### Irregular Boundaries and Different Log Densities

First, log density is assumed to be uniform over the entire area. Figures for total area and average skid distance of the skidding area shown in figure 1 are computed by the hand-held caluclutor and compared to results from the digitizer-computer program:

| | Computer-digitizer | Hand-held calculator |
|---|---|---|
| Total area | 50.00 acre | 50.51 acre |
| Aver. skid distance | 1,012 feet | 1,021 feet |

Next, consider the three areas with different log density shown in figure 1, and compute acreage and average skid distance (ASD) for each area with the programable calculator:

| Area, i | Area log density, $\delta_i$ logs/acre | Area, $A_i$ acres | ASD, $r_i$ feet |
|---|---|---|---|
| 1 | 400 | 17.25 | 1,466.0 |
| 2 | 100 | 22.87 | 697.7 |
| 3 | 200 | 10.39 | 992.3 |

Average skid distance for the entire cutting unit with three areas of differing log density is 1,222.6 feet (appendix 2).

For the entire area, neglecting different log densities, computed results from the digitizer-computer method and the hand-held programable calculator are similar. When the area is divided into three log density areas with the highest log density far from the landing in area 1, the average skid distance increases about 200 feet, showing the effect of considering different log densities.

| Point number | Coordinates x | y |
|---|---|---|
| 1 | 0, | 0 |
| 2 | 150, | -100 |
| 3 | 450, | 300 |
| 4 | 850, | 100 |
| 5 | 1,250, | 400 |
| 6 | 950, | 750 |
| 7 | 900, | 1,050 |
| 8 | 1,150, | 1,400 |
| 9 | 1,000, | 1,550 |
| 10 | 600, | 1,600 |
| 11 | 250, | 1,750 |
| 12 | -200, | 1,550 |
| 13 | -600, | 1,500 |
| 14 | 200, | 1,200 |
| 15 | 300, | 1,050 |
| 16 | -100, | 750 |
| 17 | -400, | 950 |
| 18 | -900, | 1,200 |
| 19 | -1,100, | 1,150 |
| 20 | -1,100, | 1,000 |
| 21 | -950, | 800 |
| 22 | -350, | 250 |

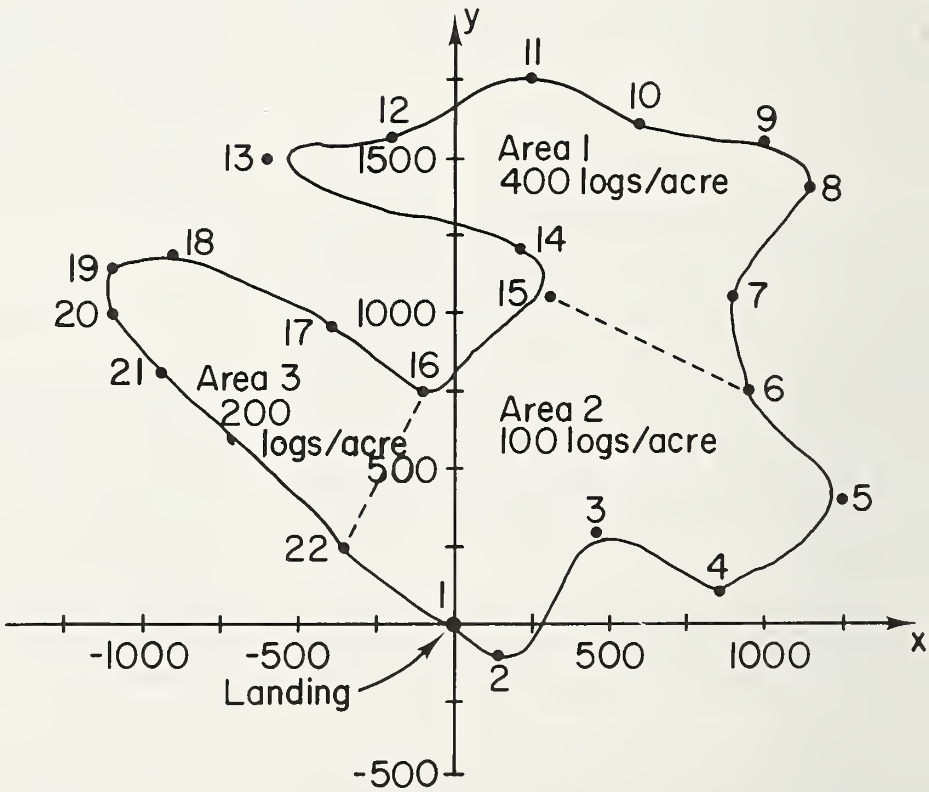


Figure 1.–Irregular boundary skidding area with three subareas of differing log density. Coordinate values are in feet. (Based on Peters and Burke 1972, p. 8)

In contrast, suppose the log density in area 2 nearest the landing is 400 logs/acre and the density in area 1 far from the landing is 100 logs/acre, the average skid distance would then be 847 feet, about 160 feet shorter than average skid distance computed without regard for log density.

No effect due to slope was considered in the previous computations. If the skidding area is generally level, these skid distances would be approximately correct. However, when average overall slope is 32% or greater, the actual skid distance on the ground is 5% or larger than the computed horizontal distance neglecting slope. These figures give some indication of slope effects on accuracy. Since most tractor logging is done on slopes less than 32%, computed average skid distance will be accurate within 5% of the value for flat ground in these instances. For any slope, but especially those greater than 32%, average yarding distance can be corrected for slope as follows:

$$\text{ASD (corrected)} = \frac{\text{ASD (horizontal)}}{\cos[\tan^{-1}(\text{percent slope}/100)]}$$

Another use for the methods described here is to compute the components of average skid distance where logs are skidded to one or more in-woods swing decks before being forwarded to the main concentration landing. Average skid distance to each respective swing deck is computed just as previously described. If logs are uniformly distributed only area is considered; but if one or more separate log densities are found, then both area and variable log density are taken into account. Average skid distance from a swing deck to the central landing is the same as the distance between the two points because all logs are concentrated at the swing landing before forwarding.

## Discussion

### Logs Carried Per Turn

One of the assumptions underlying the previous analysis is that the average number of logs carried on each turn can be defined. In the derivation in appendix 2, this assumption allows the expression for average skid distance to be independent of number of logs carried per turn. This assumption is somewhat easier to justify for cable choker skidders than for grapple skidders.

For cable skidders, average logs per turn is a function of number of chokers carried, of average log size, of log dispersion and orientation with respect to the skid trail, and of operator choice. For a given area where these variables can be defined, average logs per turn can also be estimated.

For grapple skidders, all factors above, except number of chokers, determine number of logs carried per turn. In addition, grapple size will affect the number of logs carried.

The analysis in appendix 2 is also valid if weight per acre, $w_{ai}$ or volume per acre, $v_{ai}$, is substituted for logs per acre, $\delta_i$. For tractor skidding, especially grapple skidders, and for cable yarding, weight is the limiting factor on load size for a given set of circumstances. Weight can be approached indirectly by using the species weight per cubic foot and cubic volume per acre in the derivations of appendix 2, and the computer procedures of appendix 1.

### Wander Factor

Tracked or rubber tired skidders seldom travel in a straight line from load point to deck. Actual distance traveled is usually greater than the computed straight-line average skid distance. Thus, the computed average skid distance can be multiplied by a wander factor [3] to better estimate distance skidded. However, the wander factor will not change the effect of variable log density. If an area of high log density is located far from the deck, average skid distance will be greater than if logs were uniformly distributed over the entire skid area. If a wander factor is not known, then the best that can be done is to compute a straight-line skid distance, knowing that actual skid distance will be greater by some unknown amount.

### Application

The average skid distance computed by the method described here is simply the average one-way distance, weighted by number of logs per area, that is traveled by a skidder or cable yarder carriage. Not included in the computation are distances traveled to attach chokers from a skidder equipped with a winch cable or lateral distances traveled to attach chokers from a cable yarder equipped with a skid line.

A survey of timber sale appraisal techniques indicates that external and average yarding distance

[3] *Wander factor is the ratio of actual travel distance to straight line distance.*

are key factors in computing estimated yarding cost. Lateral yarding distance, however, is not generally included with average yarding distance, and consequently, was not used in this analysis.

## Summary

When individual landings and the area each serves can be identified as part of timber sale planning, methods presented here can help compute average skid distance. In addition, areas of different known log densities can be included in the analysis. Effects of differing log densities on average skid distance can be significant.

The interested user can combine these examples with theory presented in the appendices to alter the method for his own application. These computations may also be programed on other computers, including large scale digital computers.

## Literature Cited

Lysons, Hilton H., and Charles N. Mann. 1965. Correction of average yarding distance factor for circular settings. U.S. For. Serv. Res. Note PNW-24, 3 p., illus. Northwest For. and Range Exp. Stn., Portland, Oreg.

Matthews, Donald M. 1942. Cost control in the logging industry. 374 p. McGraw-Hill, Inc., N.Y.

Peters, Penn A., and J. Doyle Burke. 1972. Average yarding distance on irregular-shaped timber harvest settings. USDA For. Serv. Res. Note PNW-178, 13 p., illus. Pac. Northwest For. and Range Exp. Stn., Portland, Oreg.

Suddarth, Stanley K., and Allyn M. Herrick. 1964. Average skidding distance for theoretical analysis of logging costs. Res. Bull. No. 789, 6 p., illus. Purdue Univ. Agric. Exp. Stn., Lafayette, Ind.

# Appendix 1

## Average Skid Distance Computations

### A Numerical Approach

The essential feature of this numerical method is that an area with any shape can be bordered and approximated by a series of connected straight line segments. Lines drawn from the ends of each such line segment to the coordinate origin form a triangle which is the basic area for subsequent computation (fig. 2).

Consider for the moment, only the single triangle highlighted in figure 2. Average skid distance for this triangle is the distance, $r$ from the origin of coordinates, 0, to the triangle's polar centroid, P.C., assuming uniform density within the triangle.

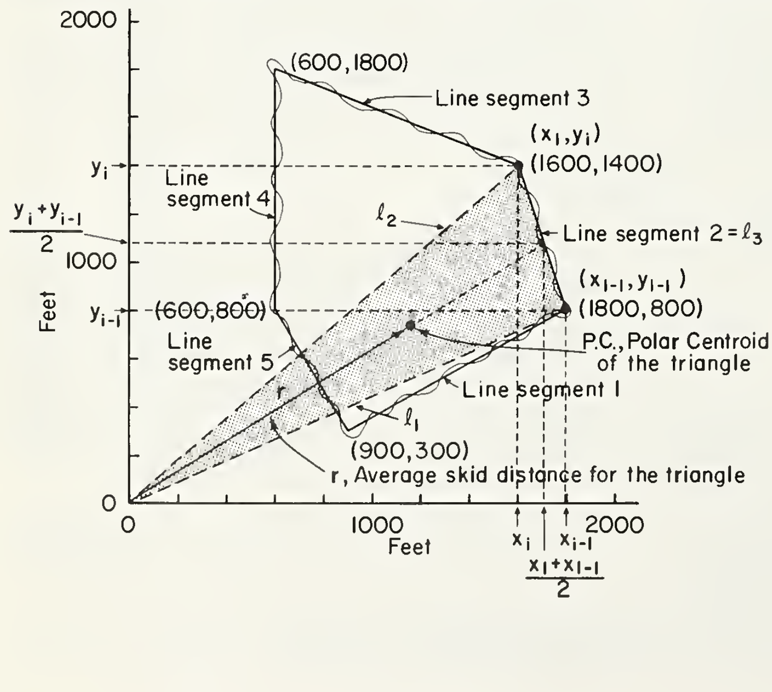


**Figure 2.—Irregular area boundary approximated by a polygon. The shaded triangle is the basic area for computation of average skid distance (ASD) and area.**

The exact formula for distance, $\mathscr{r}$, is

$$\frac{\ell_1 + \ell_2}{6\ell_3^2}\left(\ell_3^2 + (\ell_1 - \ell_2)^2\right)$$

$$+ \frac{[\ell_3^2 - (\ell_1 - \ell_2)^2]\ [(\ell_1 + \ell_2)^2 - \ell_3^2]}{12\ell_2^3}$$

$$\log_e \frac{\ell_1 + \ell_2 + \ell_3}{\ell_1 + \ell_2 - \ell_3}$$

where $\ell_1^2 = x_{i-1}^2 + y_{i-1}^2$

$\ell_2^2 = x_i^2 + y_i^2$

and $\ell_3^2 = (x_i - x_{i-1})^2 + (y_i - y_{i-1})^2$

are the lengths of the triangle sides, and where the coordinates, $(x_i, y_i)$ and $(x_{i-1}, y_{i-1})$, represent the two non-zero verticles of each computing triangle as shown in figure 2[4]. Details of the derivation can be obtained from the author.

An alternative derivation is based on approximating the distance, $\mathscr{r}$, by the expression

$$AYD = \mathscr{r} \simeq 2/3 \left[\left(\frac{x_i + x_{i-1}}{2}\right)^2 + \left(\frac{y_i + y_{i-1}}{2}\right)^2\right]^{1/2}$$

$$AYD = \mathscr{r} \simeq 1/3 \left[\left(x_i + x_{i-1}\right)^2 + \left(y_i + y_{i-1}\right)^2\right]^{1/2}$$

This approximate formula requires one vertex of the triangle be located at the origin, o. The approximation improves as the angle of the triangle touching the origin decreases.

The triangle area, $A_i$, is defined by coordinates of the line segment endpoints as

$$A_i = 1/2\,(x_i y_{i-1} - x_{i-1} y_i)$$

The sign of $A_i$ and of the sum of all $A_i$'s depends on the sequence of data entry. For example, counter-clockwise data entry in figure 2 results in an area with a negative sign. This is not an error, but a result of data entry sequence. If data for this example were entered in clockwise sequence, the resulting area is positive. The absolute value function in the calculator program is applied to the final value for area and insures positive area consistent with expectation.

The average skid distance for the entire area shown in figure 2 is

$$ASD = \frac{\sum_{i=1}^{n} A_i \mathscr{r}_i}{\sum_{i=1}^{n} A_i} \qquad i = 1, 2, \cdots, \text{no. of boundary line segments.}$$

Table 1 illustrates the mechanics of computation for average skid distance.

As stated earlier, the minus sign in the column total for triangle area and in the column for polar moment results from the counter-clockwise direction of line segment numbering in figure 2.

Table 1.—Illustration of mechanical computations for obtaining average skid distance

| Triangle (line segment) number, i | Polar coordinate $r_i$ | Triangle area $A_i$ | First polar moment, $r_iA_i$ |
|---|---|---|---|
| 1 | 971.83 | $-90 \times 10^3$ | $-0.8746 \times 10^8$ |
| 2 | 1,349.90 | $-620 \times 10^3$ | $-8.3694 \times 10^8$ |
| 3 | 1,294.43 | $-1{,}020 \times 10^3$ | $-13.2032 \times 10^8$ |
| 4 | 954.52 | $+300 \times 10^3$ | $+2.8636 \times 10^8$ |
| 5 | 620.04 | $+270 \times 10^3$ | $+1.6741 \times 10^8$ |
| Total | | $-1{,}160 \times 10^3$ | $-17.9095 \times 10^8$ |

$$ASD = \frac{-17.9095 \times 10^8}{-1{,}160 \times 10^3} = 1{,}543.9$$

$$\text{Area} = \left|\frac{-1{,}160 \times 10^3 \text{ sq. ft.}}{43{,}560 \text{ sq. ft./acre}}\right| = 26.63 \text{ acres}$$

## Program for a Hand-Held Computer

The preceding algorithm is programed on the Hewlet-Packard 65, a hand-held programable calculator[5]. Figures 3 and 4 show the coding and operating steps.

[4] *Personal communication from P.A. Peters, Associate Professor, Forest Engineering Department, Oregon State Univ., Corvallis, Oreg., 1976.*

[5] *The use of trade, firm, or corporation names in this publication is not an official endorsement or approval by the U.S. Department of Agriculture of any product to the exclusion of others which may be suitable.*

# HP-65 Program Form

Title: Skid distance — irregular area  Page ___ of ___

SWITCH TO W/PRGM. PRESS [f] [PRGM] TO CLEAR MEMORY.

| KEY ENTRY | CODE SHOWN | COMMENTS |
|---|---|---|
| LBL | 23 | Key $x_0$, ENTER, |
| A | 11 | $y_0$, A |
| LBL | 23 | |
| 1 | 01 | |
| STO 2 | 33 02 | Put $y_0$ in R2 |
| g R↓ | 35 08 | |
| STO 1 | 33 01 | Put $x_0$ in R1 |
| g R↑ | 35 09 | Put last x & y coordin. |
| g x⇄y | 35 07 | in x, y registers |
| R/S | 84 | Key $x_i$, R/S |
| ENTER↑ | 41 | |
| R/S | 84 | Key $y_i$, R/S |
| STO 4 | 33 04 | Put $y_i$ in R4 |
| g R↓ | 35 08 | |
| STO 3 | 33 03 | Put $x_i$ in R3 |
| RCL 1 | 34 01 | Recall $x_{i-1}$ |
| + | 61 | $x_i + x_{i-1}$ |
| $f^{-1}$ | 32 | $(x_i + x_{i-1})^2$ |
| $\sqrt{x}$ | 09 | |
| RCL 4 | 34 04 | Recall $y_i$ |
| RCL 2 | 34 02 | Recall $y_{i-1}$ |
| + | 61 | $(y_i + y_{i-1})$ |
| $f^{-1}$ | 32 | $(y_i + y_{i-1})^2$ |
| $\sqrt{x}$ | 09 | |
| + | 61 | $(x + x_{i-1})^2 + (y_i + y_{i-1})^2$ |
| f | 31 | $[(x_i + x_{i-1}) + (y_i + y_{i-1})^2]$ ½ |
| $\sqrt{x}$ | 09 | |
| 3 | 03 | ⅓ $[(x_i + x_{i-1})^2 + (y_i + y_{i-1})^2]$ ½ |
| ÷ | 81 | |
| STO 7 | 33 07 | Store $r_i$ in R7 |
| RCL 3 | 34 03 | Recall $x_i$ |
| RCL 2 | 34 02 | Recall $y_{i-1}$ |
| X | 71 | $x_i y_{i-1}$ |
| RCL 1 | 34 01 | Recall $x_{i-1}$ |
| RCL 4 | 34 04 | Recall $y_i$ |
| X | 71 | $x_{i-1} y_i$ |
| — | 51 | $(x_i y_{i-1} - x_{i-1} y_i)$ |
| 2 | 02 | $A_i = ½ (x_i y_{i-1} - x_{i-1} y_i)$ |
| ÷ | 81 | |
| STO 8 | 33 08 | Store $A_i$ in R8 |
| RCL 5 | 34 05 | Recall current value $\Sigma A_i$ |
| + | 61 | Add new $A_i$ value |
| STO 5 | 33 05 | Store new cur. value $\Sigma A_i$ |
| RCL 7 | 34 07 | Recall $r_i$ |
| RCL 8 | 34 08 | Recall $A_i$ |
| X | 71 | $r_i A_i$ |
| RCL 6 | 34 06 | Recall cur. value $\Sigma r_i A_i$ |
| + | 61 | Add new $r_i A_i$ value |
| STO 6 | 33 06 | Store new current value $\Sigma r_i A_i$ |
| RCL 3 | 34 03 | Recall $x_i$ |

| KEY ENTRY | CODE SHOWN | COMMENTS |
|---|---|---|
| RCL 4 | 34 04 | Recall $y_i$ |
| GTO | 22 | Go to |
| 1 | 01 | Label 1 |
| LBL | 23 | To get ASD, key B |
| B | 12 | |
| RCL 5 | 34 05 | RECALL $\Sigma A_i$ |
| ENTER↑ | 41 | Get $\Sigma A_i$ in X & Y regs. |
| RCL 6 | 34 06 | Recall $\Sigma r_i A_i$ |
| g x⇄y | 35 07 | Switch $\Sigma r_i A_i$ & $\Sigma A_i$ |
| ÷ | 81 | ASD = $\Sigma r_i A_i / \Sigma A_i$ |
| RTN | 24 | |
| LBL | 23 | To get area, key C |
| C | 13 | |
| RCL 5 | 34 05 | Recall area, $\Sigma A_i$ |
| g ABS | 35 06 | Obtain absolute value of $\Sigma A_i$ |
| 4 | 04 | |
| 3 | 03 | |
| 5 | 05 | Divide by 43,560 ft $^2$/ac |
| 6 | 06 | to get acres in skid |
| 0 | 00 | area |
| ÷ | 81 | |
| RTN | 24 | |

Figure 3.—Program coding for the average skid distance program computed on an HP-65 programmable calculator.

| REGISTERS | |
|---|---|
| R1 | $x_{i-1}$ |
| R2 | $y_{i-1}$ |
| R3 | $x_i$ |
| R4 | $y_i$ |
| R5 | $\Sigma A_i$ |
| R6 | $\Sigma r_i A_i$ |
| R7 | $r_i$ |
| R8 | $A_i$ |
| R9 | |

| LABELS | |
|---|---|
| A | Initialize |
| B | Aver. skid dist. |
| C | Skid Area |
| D | |
| E | |
| 0 | |
| 1 | Compute |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |

| FLAGS | |
|---|---|
| 1 | |
| 2 | |

9320-0616

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM.

# HP-65 User Instructions

Title Skid distance — irregular area Page ____ of ____

Programmer D. Donnelly Date ____

| $x_o$ $y_o$ | ASD | skid area | | |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| | | | | |

| STEP | INSTRUCTIONS | INPUT DATA/UNITS | KEYS | | OUTPUT DATA/UNITS |
|---|---|---|---|---|---|
| 1 | Key first x coord., $x_o$; Key ENTER↑ | | no. | ENTER↑ | |
| 2 | Key first y coord., $y_o$; Key A | | no. | A | |
| 3 | When program stops w/$x_o$ in display, | | | | |
| 4 | Key next x coord., $x_i$; Key R/S | | no. | R/S | |
| 5 | Key next y coord., $y_i$; Key R/S | | no. | R/S | |
| 6 | Program stops with last x coord. entered in | | | | |
| | display; | | | | |
| | If more coordinates to input, go to #4 | | | | |
| | When closure[1] is achieved and processed, | | | | |
| | go to 7 | | | | |
| | | | | | |
| 7 | Key RTN, B | | RTN | B | Aver. skid dist., ft |
| 8 | Key C | | C | | Skid area, ac |
| | | | | | |
| 9 | To start a new area, Key f STK , f REG , | | f | STK | |
| | RTN | | f | REG | |
| | | | RTN | | |
| 10 | Go to Step 1 | | | | |
| | | | | | |
| | [1]*Closure is achieved when all coordinates have been entered, including, at the end, the values of $x_o$ & $y_o$; ie., $x_o$ & $y_o$ are entered twice, once at the start and once at the end.* | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

**Figure 4.—Operating instructions for the average skid distance program computed on an HP-65 programmable calculator.**

Two operational points should be noted. The actual periphery of an area is more closely estimated by increasing the number of approximating line segments. At some point, though, increased accuracy from additional line segments is offset by increased computational time. Second, since we are simulating a continuous curve by a series of straight line segments, careful placement of a given number of line segments in relation to the area boundary can increase accuracy. Sharp changes in the boundary direction are good places to start or end a straight segment.

The interested reader can use figure 1 as an exercise and compute the results previously given in Irregular Boundaries and Different Log Densities using the program code and operating instructions.

## Appendix 2
## Variable Log Density

### One Possible Derivation

Most practical skidding areas will have no more than two or three different log densities. However, a theoretical background should provide for the general case and then be adapted to specific instances. First consider the general case of variable log density.

Assume a skidding area, A, is made of n subareas, $A_i$, of differing log density, (fig. 5). The number of turns required to skid all logs in area $A_i$ is the number of logs in $A_i$ divided by the average number of logs carried per turn.

$$t_i = \frac{l_i}{p_t}$$

where $t_i$ = number of turns required to skid all logs in area, $A_i$.
$l_i$ = number of logs in area, $A_i$.
$p_t$ = average or estimated number of pieces (logs) carried per turn, and is assumed constant in all areas.

The number of logs in $A_i$ is the density per acre (or other unit of area) muliplied by the area.

$$l_i = \delta_i A_i$$

where: $\delta_i$ = density in logs per unit area (acre).

Total distance traveled by skidding all logs in area $A_i$ is the number of turns multiplied by the AYD of the subarea, the distance between the landing and polar centroid of $A_i$.

$$d_i = t_i \, AYD_i = t_i r_i$$

where $d_i$ = distance traveled skidding all logs in area $A_i$.
$r_i = AYD_i$ = distance from the landing to the polar centroid of area, $A_i$.

Total skidding distance for all the subareas, $A_i$, is found by summing $d_i$ for all i areas.

$$D = \sum_{i=1}^{n} d_i.$$

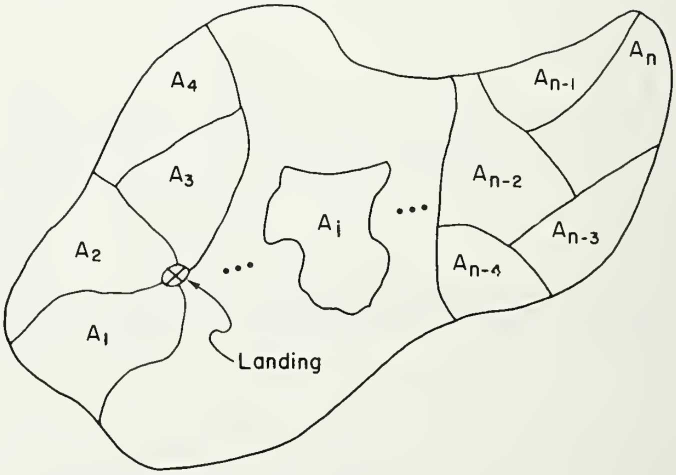


**Figure 5.—Skidding area, A, with n subareas, $A_i$, i=1, 2, ... n.**

Similarly, total turns required to skid all logs from the entire area A are

$$T = \sum_{i=1}^{n} t_i.$$

Average skid distance is total distance travelled skidding divided by total turns,

$$ASD = \frac{D}{T} = \frac{\sum_{i=1}^{n} d_i}{\sum_{i=1}^{n} t_i}$$

Substituting all previously derived quantities gives

$$ASD = \frac{\sum_{i=1}^{n} t_i r_i}{\sum_{i=1}^{n} t_i}$$

$$ASD = \frac{\sum_{i=1}^{n} \frac{l_i}{p_t} r_i}{\sum_{i=1}^{n} \frac{l_i}{p_t}}$$

$$ASD = \frac{\sum_{i=1}^{n} \delta_i A_i r_i}{\sum_{i=1}^{n} \delta_i A_i}$$

Note that average or estimate number of logs carried per turn, $p_t$, cancels and is not a factor in the remaining derivation. This is allowed only when estimated number of logs carried per turn can be assumed to be constant, as discussed earlier in the report.

If we let the number of areas $A_i$ in A become infinitely large, we get the following general integral form for average skid distance:

$$ASD = \frac{\int_A \delta(a)\, r(a)\, da}{\int_A \delta(a)\, da}$$

where $\delta$ and r are functions of the area involved. To summarize, the general formula for average skid distance depends on the areal variation of log density. $\delta$, and on the distance, r, from deck to log location. Shape of the area is not a factor in this generalized formula but is considered when applying the method to a specific skidding example, shown below.

## Average Skid Distance Formula — Circular Sector

To illustrate the effect of variable log density, apply the derivation results to a circle sector used as an example by Suddarth and Herrick (1964). In addition, partition the circle sector into concentric zones of differing log density (fig. 6). This configuration, for example, might approximate a wedge shaped cable logging setting with different cutting densities. For a small differential area, da, in any zone

$$da = r\, dr\, d\Theta.$$

$$\text{For all n zones, } ASD = \frac{\sum_{i=1}^{n} \int_0^{k\pi} \int_{R_{i-1}}^{R_i} \delta_i r_i^2\, dr_i\, d\Theta}{\sum_{i=1}^{n} \int_0^{k\pi} \int_{R_{i-1}}^{R_i} \delta_i r_i\, dr_i\, d\Theta}$$

Both summation and integration are necessary here because log density, $\delta$, is not a continuous function over the range of values for r. In fact, for this example, $\delta$ is considered constant within each log density zone, just like a skidding area with several cutting intensities.

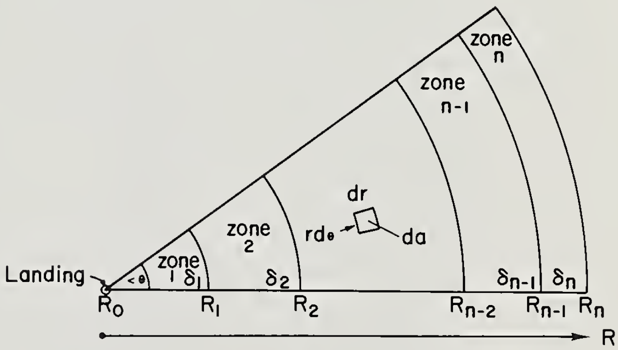


**Figure 6. —Circular sector with n concentric log density zones and with radius R.**

Proceeding with the evaluation of this formula gives

$$ASD = \frac{\sum_{i=1}^{n} \delta_i \int_0^{k\pi} \frac{r_i^3}{3}\Big]_{R_{i-1}}^{R_i} d\Theta}{\sum_{i=1}^{n} \delta_i \int_0^{k\pi} \frac{r_i^2}{2}\Big]_{R_{i-1}}^{R_i} d\Theta}$$

$$ASD = \frac{\frac{1}{3} \sum_{i=1}^{n} \delta_i \left(R_i^3 - R_{i-1}^3\right) \int_0^{k\pi} d\Theta}{\frac{1}{2} \sum_{i=1}^{n} \delta_i \left(R_i^2 - R_{i-1}^2\right) \int_0^{k\pi} d\Theta}$$

$$ASD = \frac{2}{3} \frac{\sum_{i=1}^{n} \delta_i \left(R_i^3 - R_{i-1}^3\right)}{\sum_{i=1}^{n} \delta_i \left(R_i^2 - R_{i-1}^2\right)}$$

This is the formula for average skid distance for a circle or circle sector with concentric zones of varying log density. Note three points. In figure 6, $R_0 = 0$, but this is not a requirement; $R_0$ can be greater than zero. Also, we do not have to include all bands; for example, density, $\delta_2$, could be zero. Finally, when all log densities are equal, the summations for average skid distance reduce to ASD = 2/3R, the same result for a circular segment computed in research cited earlier.

Now apply the general formula for a circular sector skidding area to a specific example, with three concentric zones of different log density, as shown below:

| Zone, i | Zone log density, $\delta_i$, logs/acre | Zone inner distance from landing, $R_{i-1}$, | Zone outer distance from landing, $R_i$, |
|---|---|---|---|
| 1 | 100 | 0 | 200 |
| 2 | 50 | 200 | 400 |
| 3 | 300 | 400 | 1,000 |

Substituting these numbers into the average skid distance formula for a circle or circular segment gives an average skid distance of 724 feet. Average skid distance is 667 feet when variable log density is neglected.

As noted earlier, the general integral form for average skid distance can theoretically apply to areas of any shape. Unfortunately, direct integral evaluation requires that log densities and the equations defining area boundaries be well defined, mathematically speaking. Thus, evaluation of definite integrals is relatively easy for circles, circular segments, and rectangles. However, integrals for irregular shapes are difficult, if not impossible, to evaluate and are more efficiently evaluated by approximate numerical methods as illustrated in this report.

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Donnelly, Dennis M. 1978. Computing average skidding distance for logging areas with irregular boundaries and variable log density. Gen. Tech. Rep. RM-58, 10 p. Rocky Mt. For. and Range Exp. Stn., For. Serv., U.S. Dep. Agric., Fort Collins, Colo. 80526.

Current silvicultural practices may result in different cutting intensities on small adjacent areas, resulting in different log densities over the log skidding area. Previous theory on skid distance computation is revised in terms of irregular skid area boundaries and variable log density. A program for a hand-held calculator is presented to compute average skid distance for irregular shaped areas. Methods presented apply either to tractor skidding or to cable yarding distances.

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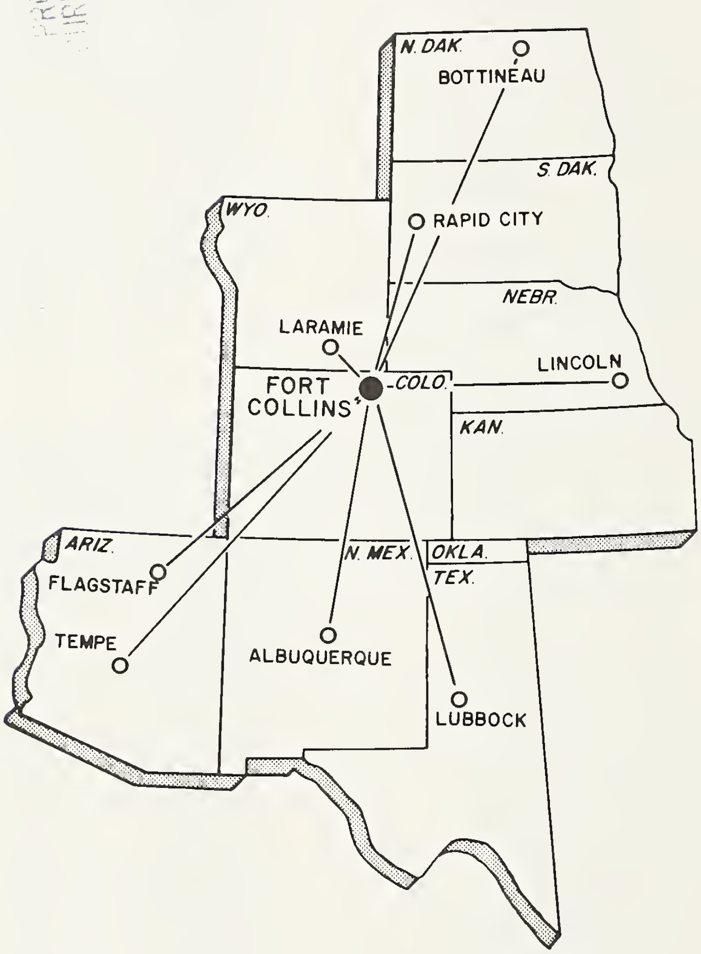
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